Toolkit Functions I

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1 Administrivia

Announcements

Assignment

No new reading, but another online quiz on 2.3.

From Last Time

Characteristics of functions.

2 Introduction

Keys to understanding toolkit functions:

1. Behavior around the origin.

2. Above/below line $y = x$.

3. Symmetry: even/odd.

4. Asymptotes.
Functions and terminology:

1. Constant function.
   \[ f(x) = \pi. \]

2. Linear function.
   Identity function.

3. Quadratic function.
   \[ y = x^2 \]

   \[ f(x) = x^3 \]
   Steeper slope than quadratic. Odd symmetry. Concave up and down.

5. Square root function.
   \[ y = \sqrt{x} \] Equivalent to \( x^{1/2} \).
   Calculator warning: Use parentheses — \( \sqrt{(x + 5)} \sqrt{(2x)} \).
   Principal (positive) square root — otherwise, no function. But, we must remember when we have \( x^2 = 9 \) that \( \sqrt{x^2} = \sqrt{9}, x = \pm 3 \).
   Concave down.

6. Exponential function.
   \[ y = 2^x \]
   Concave up. Horizontal asymptote at \( y = 0 \).

7. Reciprocal function.
   \[ f(x) = \frac{1}{x} \] Equivalent to \( x^{-1} \).
   Vertical asymptote at \( x = 0 \). Odd symmetry. Also has horizontal asymptote at \( y = 0 \).
8. Sine function

\[ y = \sin(x) \]

Period is \(2\pi\). Turning points at \(\frac{\pi}{2}, \frac{3\pi}{2}\), etc. Zeroes at 0, \(\pi\), \(2\pi\), etc.

9. Absolute value function

\[ f(x) = |x| \]

Finding the \texttt{ABS} key on the calculator — three keys below \texttt{2nd} key. Example of a piecewise-defined function:

\[ |x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases} \]

(How do I read one of these things???)

3 Class Practice

Use your calculator to graph each of the toolkit functions, using a window of \(-4 \leq x \leq 4\) and \(-4 \leq y \leq 4\). For each function, answer these questions:

1. Identify the domain and range of the function.
2. For what part of the domain is the function above the line \(y = x\) below?
3. What symmetry does the function have, if any? What are its asymptotes, if any?

Pg. 69: 1, 3.