Geometric Growth Models

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1 Administrivia

Announcements

Collect homework.

Exam Friday. Standard procedure: Review question quiz for Thursday’s review. One page of notes allowed.

Assignment

Read 3.5. Online quiz.


From Last Time

Recursive functions.

2 A Program for Recursive Systems

See handout.
3 Introduction

1. Geometric growth: Value at time $n$ is directly proportional to value at time $n - 1$.

$$t_n = (1 + k)t_{n-1}$$

$k > 0$: growth; $k < 0$: decay.

2. Defn. of geometric progression:

   A sequence (as 1, 1/2, 1/4) in which the ratio of a term to its predecessor is always the same (hence, a constant).

Compare arithmetic progression.

3.1 Examples and Closed Form

1. Blue jeans fade when they are washed: they lost 2% of their color. How much of the original color is left after 50 washes?

   Use 1.0.

   Growth or decay?

2. Electrical power consumption increases at a rate of 5% per year. This year 500 MW were used. How long before consumption doubles?

   Growth or decay?

3.1.1 Closed Form Models

1. Consider the blue jeans example:

   $$D_0 = 1.0$$
   $$D_1 = 0.98 \times 1.0 = 0.98$$
   $$D_2 = 0.98 \times 0.98 = 0.98^2$$
   $$D_3 = 0.98 \times 0.98^2 = 0.98^3$$
   $$D_4 = 0.98^4$$
   $$D_n = ???$$

   So: $y = f(n) = 0.98^n$. 
2. Power consumption model. What’s the closed form?

\[ f(x) = 1.05^x \times 500. \]

Can we determine more precisely when the consumption doubles? \( x = 14.2067 \) years. Graph \( y = 1.05^x \) and \( y = 2 \) and find the intersection.

Another explicit model: Power usage doubles in 14.2 years. Continuous growth. General model is \( f(t) = 500 \times 2^{kt} \), where \( k \) is a constant and \( t \) is time in years. Knowing that power usage doubles in 14.2 years, solve for \( k \):

\[ 1000 = 500 \times 2^{14.2k} \]

Solution:

\[ f(t) = 500 \times 2^{14.2t} \]

### 3.2 Class Exercise